

Some Lower Bounds for Hales-Jewett Numbers

Eric Tressler

A lower bound below will look like:

$HJ(3, 2) > 2$:

001010100

This gives a 2-coloring (in the colors 0 and 1) of the elements of $\{0, 1, 2\}^2$ listed in lexicographic order. That is, 00 is colored 0, 01 is colored 0, 02 is colored 1, 10 is colored 0, and so on. The claim is that this coloring contains no monochromatic combinatorial lines (and of course it doesn't, as can be checked by hand).

Checking the examples below is best left to a computer.

$HJ(3, 3) > 6$:

12001202002212121201012201020201121212121101020002002102122000221212202001220110
20212122212020211211212002000221222102111100222120222202121001121222012011102201
01010220012221112210012121202220212120112020200120200011002121201210020011100001
21212111221221020110120201002210101002001102210022021112121210201000222122112022
10112022021200221211000102202212110201010202000010212110101201012211002002112010
12122122100200110200221210022010022121201202201011001002122110220011221012122100
22110001012102012221021202210220121110202020202021102122001002222121211000110010
21102020020202010212100101100011101222101122201011012111100021102112002010120121
02121022212101212110002100202112201110200001001022112100011112100110002101121002
211022221

$HJ(4, 2) > 6$:

00110010100011010110010100011110010111101001010111010100011010000010011001011010
01011100101010011100101001100111110010110001010101001100111000011011011010000110
10101000011100100001000101101011110110110110110001000010110110010001111101101
10110101101000100010011010000101010111100011100010101000010100111101101101001010
01011101001011001110110110010010001010011010110010100010010001111010000111010011
010100100010110001010011011111001111001110100110101110000110100011001011100001
01101100100110111001100100110100001001111101000111000100001101000001110101010110
01111011110010110100100110110101101010111011010110110010011000010110010100011010

10011100011011000011111001001001110000111110101111001010100101000100010110000110
10000110010101111101110000111010001110000110110011011000001111100011101011101101
10010001111010111000011110010100100111010010110000010100011110000010011011010011
11011010101101110001011110101011101010011101001011100100100001011010100101010111
11100010110100101100100100111010101111100100010001001011010010110110101000010001
01101000001010111001000101101010101011101000010101100101100100101101100110100101
01101010100000111000000101111101111000011000011110000011001101001001001111100101
00111110010110000001001001111000001001010101110011011100000100110111010010101110
01011101001100101001100001101001011000111000000101001001000101110101100111101110
10001110010100100010010110010101001100100111100010011010001011000010011100111100
10100100100010011100010011010010011000010101101111011000100000101001111001110110
000110100100110100101001101001111010011101111001100011111001000001011000101000
10010010001011000110010100011010011111001010110101101101110100101010011111011001
01000001100000110001110000111010010110010010011101001000101010110001110010110001
00101001011001001110011001011001100001100100101110000001110100100010101000111101
01111010010101110011001011100101001000101101101110011101010101100111001110001001
01111010101101001011110011010111011110010010101010000001111010001010010111010101
010110110010010101000010101110100001010110100101010100110000101001101110010100
00100101101100110111110100101010110110111010011101000010001110101011110101110100
11001010010100110001100000111100100001100100100110110011111010000111011010010110
10101101000101100110100001001001010101101000001010011001011001010010100110110100
0110100000110110011111010100100011010010011010010010011001101101101010001010101
11010010100110111001011111000010100101100101101100110001101011010111110010111010
10101101000101101101001001011000000101101010000100101000001101010101011110100010
10111010011010010010011111011001110010000111101110001011101101000010110101010001
10110001100001010111010010101011000101111010111011100101101100100101100000110011
11101011010001000010100001001101100111001010010011011011001111100010101001010011
01000111110100101001010110000111111001011000011101011010001001011000001111011000
00010110010110000011110010100111111010110110110010111100001001000110011111000001
11000001000101100100001001001101000111000111101100101010001001011101011001101000
10000101110110111110010010111010110100110010000110111000011101110011110001101110
10000011110111001011101010110100101001011101111011000101010010110011010110101010
00110111011111000111110001001000010111101000110011001001100000111000000111001010
00010110101100111010100000110101110011100001011011001010011010011011000111010100
11001010101101010010010110010011101001010101001101010010011010110001011001011000
01111100100011010101110010000010101111100101001110100101100100101001001101101100
00110100101001111110101100011101110001110010100000100001111001010100110100110110
11011011010010110010111010111000010101001010110101110001110010110100111001010001
10000101000101101000101001100001001110001011011011000101001011000010100111000011
10100111000110101110101101111101101100101100100001110100110100101101001000111011
00100011110111010100111000110111100001111101110001111101001000101100011110010011
110010010010111000111000101011100110100111001000010101110111001111101000

00011110011100110101011010101010100101110110111000110101100011011110100000111010
1000010110110110